

# Supergravity gauge theories strike back: There is no crisis for SUSY but a new collider may be required for discovery

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More than 30 years ago, Arnowitt-Chamseddine-Nath (ACN) and others established the compelling framework of supergravity gauge theories (SUGRA) as a picture for the next step in beyond the Standard Model physics. We review the current SUGRA scenario in light of recent data from LHC8 collider searches and the Higgs boson discovery. While many SUSY and non-SUSY scenarios are highly disfavored or even excluded by LHC, the essential SUGRA scenario remains intact and as compelling as ever. For naturalness, some non-universality between matter and Higgs sector soft terms is required along with substantial trilinear soft terms. SUSY models with radiatively-driven naturalness (RNS) are found with high scale fine-tuning at a modest  $\sim 10\%$ . In this case, natural SUSY might be discovered at LHC13 but could also easily elude sparticle search endeavors. A linear  $e^+e^-$  collider with  $\sqrt{s} > 2m(\text{higgsino})$  is needed to provide the definitive search for the required light higgsino states which are the hallmark of natural SUSY. In the most conservative scenario, we advocate inclusion of a Peccei-Quinn sector so that dark matter is composed of a WIMP/axion admixture *i.e.* two dark matter particles.

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## INTRODUCTION

The recent amazing discovery of a Higgs scalar with mass  $m_h \simeq 125$  GeV by the Atlas[1] and CMS[2] collaborations at LHC seemingly completes the Standard Model (SM), and yet brings with it a puzzle. It was emphasized as early as 1978 by Wilson/Susskind[3] that fundamental scalar particles are unnatural in quantum field theory. In the case of the SM Higgs boson with a doublet of Higgs scalars  $\phi$  and Higgs potential given by

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (1)$$

one expects a physical Higgs boson mass value

$$m_h^2 \simeq 2\mu^2 + \delta m_h^2 \quad (2)$$

where the leading radiative correction is given by

$$\delta m_h^2 \simeq \frac{3}{4\pi^2} \left( -\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8 \cos^2 \theta_W} + \lambda \right) \Lambda^2. \quad (3)$$

In the above expression,  $\lambda_t$  is the top quark Yukawa coupling,  $g$  is the  $SU(2)$  gauge coupling and  $\lambda$  is the Higgs field quartic coupling. The quantity  $\Lambda$  is the UV energy cutoff to otherwise divergent loop integrals. Taking  $\Lambda$  as high as the reduced Planck mass  $M_P \simeq 2.4 \times 10^{18}$  GeV would require a tuning of  $\mu^2$  to 30 decimal places to maintain the measured value of  $m_h^2$ . Alternatively, naturalness—requiring that no parameter needs to be adjusted to unreasonable accuracy (as articulated by Dimopoulos and Susskind[4])—required that loop integrals be truncated at  $\Lambda \sim 1$  TeV: *i.e.* one expects the SM to occur as an effective field theory valid below  $\sim 1$  TeV, and that at higher energies new degrees of freedom will be required. While the technicolor route[4, 5] banished

all fundamental scalars from the theory, an attractive alternative which naturally admitted fundamental scalars—supersymmetry—was already emerging.

In supersymmetry, the fundamental bose-fermi spacetime symmetry guaranteed cancellation of all quadratic scalar mass divergences so that scalar fields could coexist with their well-behaved fermion and gauge-boson brethren. Early models based on global SUSY could be seen to lead to phenomenological inconsistencies: some superpartners would have to exist with masses below their SM partners: such a situation—*e.g.* the presence of scalar electrons with mass less than an electron—would not have eluded experimental detection. The simultaneous development of models based on gauged, or local SUSY, provided a path forward which was consistent with phenomenological requirements. Local SUSY models—where the spinorial SUSY transformation parameter  $\alpha$  in  $e^{-i\bar{\alpha}Q}$  depended explicitly in spacetime  $\alpha(x)$ —required the introduction of a gravitino-graviton supermultiplet, and hence were called supergravity theories, or SUGRA for short[6, 7]. The SUGRA sum rules for sparticle masses were modified so that all the unseen superpartner masses could be lifted up to the fundamental scale set by the gravitino mass  $m_{3/2}$ . Since these theories necessarily included gravity, they also necessarily contained non-renormalizable terms multiplied by powers of  $1/M_P$ . The modern viewpoint is then that SUGRA theories might be the low energy effective theory obtained from some more fundamental ultra-violet complete theory such as superstrings.

## SUGRA gauge theories

The starting point for construction of realistic supersymmetric models was the development of the Lagrangian for  $N = 1$  locally supersymmetric gauge theories. The final result, obtained by Cremmer *et al.* in 1982[8] is now textbook material[9, 10]. The locally supersymmetric Lagrangian for SUSY gauge theories—after elimination of all auxiliary fields and in four-component notation with a  $+, -, -, -$  metric—is written down over several pages in [10].

To construct SUGRA gauge theories[11]<sup>1</sup>, a multi-step procedure can be followed:

1. stipulate the desired gauge symmetry for the theory,
2. stipulate the super-field content consisting of chiral scalar superfields (containing spin-1/2 matter and spin-0 superpartners), the appropriate gauge superfields in accord with the gauge symmetry from step 1 (these contain massless gauge bosons and spin-1/2 gauginos) and the graviton-gravitino supermultiplet,
3. the remaining model freedom comes from stipulating the form of the holomorphic gauge kinetic function  $f_{AB}(\phi_i)$  and superpotential  $W(\phi_i)$  and the real Kahler potential  $K(\phi^\dagger, \phi)$ . In SUGRA theories, the Kähler potential and the superpotential necessarily are combined into the Kähler function  $G = K/M_P^2 + \log |W/M_P^3|^2$ .

In complete analogy to the Higgs mechanism in local gauge theories, SUGRA theories allow for a superHiggs mechanism. In the superHiggs mechanism, if one arranges for a breakdown in local SUSY, then instead of generating a physical goldstino field, the spin-1/2 goldstino is eaten by the spin-3/2 gravitino so that the gravitino gains a mass  $m_{3/2}$ . To accomodate a breakdown in SUGRA, it is necessary to introduce a “hidden sector” of fields  $h_m$ . The hidden sector serves as an arena for SUSY breaking.

Many early models invoked a very simple SUSY breaking sector. The fields were divided between an observable sector  $C_i$  and a hidden sector  $h_m$  with a separable superpotential  $W = W_{obs}(C_i) + W_{hidden}(h_m)$  and a flat Kähler metric:  $K = C_i^\dagger C_i + h_m^\dagger h_m$ . A single hidden sector field  $h$  might obey the Polonyi superpotential:

$$W_{Polonyi} = m_{hidden}^2(h + \beta). \quad (4)$$

The  $F$ -type SUSY breaking condition  $\frac{\partial W}{\partial h} + \frac{h^* W}{M_P^2} \neq 0$  is satisfied for  $\beta^2 < 4M_P^2$ . While the  $F$ -term of  $h$  gains a

VEV  $\langle F_h \rangle \sim m_{hidden}^2$ , the scalar component gains a VEV  $\langle h \rangle \sim M_P$ . The gravitino becomes massive

$$\frac{i}{2}e^{G_0/2}M_P\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \rightarrow \frac{i}{2}e^{G_0/2}M_P\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \quad (5)$$

where  $G_0$  is the VEV of  $G$ . The gravitino mass is given by

$$m_{3/2} = e^{G_0/2}M_P \sim m_{hidden}^2/M_P. \quad (6)$$

A TeV value of  $m_{3/2}$  is achieved for a hidden sector mass scale  $m_{hidden} \sim 10^{11}$  GeV.

Once the gravitino gains mass, then an amazing simplification occurs. By replacing the hidden sector fields by their VEVs and taking the flat space limit  $M_P \rightarrow \infty$  while keeping  $m_{3/2}$  fixed, one arrives at the Lagrangian of global SUSY for the visible sector fields augmented by *soft SUSY breaking terms* consisting of gaugino masses  $M$  (assuming a non-trivial form for the gauge kinetic function), scalar squared masses  $m_{\phi_i}^2$ , trilinear  $a$  and bilinear  $b$  soft terms[11]. The soft terms all turn out to be multiples of the gravitino mass  $m_{3/2}$ . For the case of the Polonyi model, then one expects

$$\begin{aligned} m_{\phi_i}^2 &= m_{3/2}^2, \\ A &= (3 - \sqrt{3})m_{3/2}, \\ B &= A - m_{3/2} = (2 - \sqrt{3})m_{3/2} \text{ while} \\ M &\sim m_{3/2} \end{aligned} \quad (7)$$

until one specifies additionally the gauge kinetic function. The universality of scalar masses and trilinears is welcome in that it allows for the super-GIM mechanism to suppress flavor violating processes while the reality of soft terms suppresses unwanted CP violation.

While the Polonyi model soft term values are intriguing, ordinarily one does not take such a toy model seriously as being indicative of the hidden sector. More general expressions for the soft terms for a general hidden sector, including a non-flat Kähler metric, have been calculated in Ref's [13–15]. The result is that: under a well-specified hidden sector, the soft SUSY breaking terms still arise as multiples of  $m_{3/2}$  although universality is not assured so that, in general, one expects both flavor and CP-violating processes to occur. Experimental limits on such processes provide constraints to SUGRA model building efforts.

In general, there may occur a multitude of hidden sector fields along with additional hidden sector gauge symmetries. In  $4 - D$  string theory, an automatic hidden sector can arise in the form of the dilaton field  $S$  and the moduli fields  $T_m$  that parametrize the size and shape of the compactification of the extra dimensions. In addition, if there are additional hidden sector gauge groups—as would arise in  $E_8 \times E_8'$  heterotic string theory—and if the additional gauge forces become strong at an intermediate scale  $\Lambda \sim 10^{13}$  GeV, then hidden sector gauginos

<sup>1</sup> For an historical review, see *e.g.* Ref. [12].

may condense[16] resulting in a breakdown of SUSY with  $m_{3/2} \sim \Lambda^3/M_P^2$ .

In spite of the daunting plethora of hidden sector possibilities, it is still possible to make progress in matching theory to experiment by appealing to *effective field theories*. In spite of our lack of knowledge of hidden sector dynamics, we may *parametrize our ignorance* by largely eschewing the hidden sector altogether and replacing it by *an adjustable set of soft SUSY breaking parameters*. As we scan over various soft parameter values, then we are effectively accounting for a wide variety of hidden sector possibilities. Under this plan, it is possible to make additional assumptions as to how the various soft terms are related to one another. For instance, one might assume universality to suppress FCNC and CP violating processes, or one might assume various GUT relations or relations amongst soft terms arising from different string theory possibilities.

### Connection to weak scale supersymmetry

It is usually assumed that the induced soft SUSY breaking terms arise at or around the reduced Planck scale  $M_P$ . Their values at lower energy scales are obtained by solving their renormalization group equations (RGEs)[17, 18]. Inspired by 1. the fact that gauge couplings unify at a scale  $m_{GUT} \simeq 2 \times 10^{16}$  GeV, and 2. that the most parsimonious effective theory below the GUT scale is the minimal supersymmetric standard model (MSSM), the soft terms are usually imposed at  $m_{GUT}$  where it may be understood that some above-the-GUT-scale running may have already occurred, perhaps in the context of some actual GUT construct[19–21].

Under the assumption that the gaugino masses unify at  $m_{GUT}$  (as they ought to if some simple GUT holds above  $m_{GUT}$  or if the gauge kinetic function has a universal dependence on hidden sector fields) then we expect

$$m(\text{bino}) \equiv M_1 \sim 0.44m_{1/2}, \quad (8)$$

$$m(\text{wino}) \equiv M_2 \sim 0.81m_{1/2}, \quad (9)$$

$$m(\text{gluino}) \equiv M_3 \sim 2.6m_{1/2}, \quad (10)$$

where  $m_{1/2}$  is the unified gaugino mass at  $Q = m_{GUT}$ . The electroweak gauginos mix with the higgsinos to yield two physical charginos  $\tilde{W}_{1,2}^\pm$  and four neutralinos  $\tilde{Z}_{1,2,3,4}$  ordered according to ascending mass. Also, the weak scale values of the squark and slepton masses are given by

$$m_{\tilde{q}}^2 \simeq m_0^2 + (5 - 6)m_{1/2}^2, \quad (11)$$

$$m_{\tilde{e}_L}^2 \simeq m_0^2 + 0.5m_{1/2}^2, \quad (12)$$

$$m_{\tilde{e}_R}^2 \simeq m_0^2 + 0.15m_{1/2}^2, \quad (13)$$

where  $m_0$  is the unified scalar mass at  $Q = m_{GUT}$ . For more precise values, including mixing effects and radia-

tive corrections[22], one may consult one of several computer codes available for SUSY mass spectra[23, 24].

A potentially tragic feature of this construct is that the soft terms which enter the scalar (Higgs) potential,  $m_{H_u}^2$  and  $m_{H_d}^2$ , are  $\sim m_{3/2}^2$  and so manifestly positive. But phenomenology dictates that the scalar potential should develop a non-zero minimum so that electroweak symmetry is properly broken:  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ . It was conjectured already in 1982 that, if the top quark mass was large enough, then radiative effects could drive exactly the right soft term  $m_{H_u}^2$  to negative values so that EW symmetry is properly broken. This radiative electroweak symmetry breaking (REWSB) could occur if the top quark mass lay in the 100-200 GeV range[25]. While such a heavy top quark seemed crazy at the time, the ultimate discovery of the top quark with mass  $m_t = 173.2 \pm 0.9$  GeV has vindicated this approach.

A final oddity in the SUGRA gauge theory construct is the allowance of a mass term in the superpotential:  $W_{MSSM} \ni \mu H_u H_d$ . Since this term is supersymmetric and not SUSY breaking, one would expect it to occur with a value  $\mu \sim M_P$ . However, for an appropriate breakdown of electroweak symmetry and to naturally develop a weak scale VEV, then  $\mu$  is required to be  $\sim M_Z$ .

There are several approaches to this so-called SUSY  $\mu$  problem. All require as a first step the imposition of some symmetry to forbid the appearance of  $\mu$  in the first place. For instance, if the Higgs multiplets carry Peccei-Quinn charges, then  $\mu$  is forbidden under the same PQ symmetry which is also needed to solve the strong CP problem. Next, one introduces extra fields to couple to the Higgs multiplets. Invoking hidden sector field(s) which couple to  $H_u H_d$  in the Kähler potential via non-renormalizable operators

$$K \ni \lambda h^\dagger H_u H_d / M_P \quad (14)$$

(where the  $F$ -term of  $h$  develops a VEV  $\langle F_h \rangle \sim m_{hidden}^2$ ) leads to a  $\mu$  term

$$\mu \sim \lambda m_{hidden}^2 / M_P \quad (15)$$

which is of order  $m_Z$  for  $m_{hidden} \sim 10^{11}$  GeV. This is the Giudice-Masiero mechanism[26]. Alternatively, coupling the Higgs fields to a visible sector singlet  $W_{NMSSM} \ni \lambda S H_u H_d$ , where  $\phi_S$  develops a weak scale VEV, then leads to the Next-to-Minimal Supersymmetric Standard Model or NMSSM[27]. A third possibility–Kim-Nilles[28], which includes the PQ strong  $CP$  solution in an intimate way– is to couple the Higgs fields to a PQ superfield  $S$  so that  $W_{DFSZ} \ni \lambda S^2 H_u H_d / M_P$ . This is the supersymmetrized version of the DFSZ axion model and leads to a  $\mu$  term,  $\mu \sim \lambda f_a^2 / M_P$ , which gives  $\mu \sim M_Z$  for an axion decay constant  $f_a \sim m_{hidden} \sim 10^{10}$  GeV.

## STATUS OF SUGRA GAUGE THEORIES

There are three indirect experimental success stories for supersymmetric models. These are indirect in that they do not involve direct confirmation of weak scale SUSY by detection of supersymmetric matter (which would be the most important way to confirm SUSY), but instead they each involve virtual contributions of supersymmetric matter to experimental observables. Had any of these three measurements turned out quite differently, then supersymmetric models would have been placed in a difficult—perhaps untenable—position.

### Experimental successes

#### *Gauge coupling unification*

The measurements of the three SM gauge couplings to high precision over the years—especially from measurements at LEP2, Tevatron and LHC—have provided perhaps the most impressive experimental support for SUSY. From the measured values of the Fermi constant  $G_F$ , the  $Z$ -boson mass  $m_Z$ , the electromagnetic coupling  $\alpha_{EM}$  and the top quark mass  $m_t$ , the  $U(1)_Y$  and  $SU(2)_L$  gauge couplings  $g_1$  and  $g_2$  can be computed at scale  $Q = M_Z$  in the  $\overline{DR}$  regularization scheme. A variety of measurements also constrain the value of  $\alpha_s \equiv g_3^2/4\pi$  at  $Q = M_Z$ . These serve as weak scale inputs to test whether the gauge couplings actually do unify as expected in a GUT theory or not. For gauge coupling RGEs in the MSSM, the couplings do indeed unify to a precision of about a few percent; in contrast, for the SM or MSSM augmented by extra non-GUT matter, then the unification fails utterly.

#### *Top quark mass and electroweak symmetry breaking*

As mentioned previously, the Higgs potential with soft scalar masses  $m_\phi^2 \sim m_{3/2}^2$  does not admit the non-zero Higgs VEV which is needed for electroweak symmetry breaking. However, the expected value  $m_{H_u,d}^2 \sim m_{3/2}^2$  is imposed at some high scale such as  $Q = M_P$  or  $m_{GUT}$  and is modified by radiative corrections. An appropriate EWSB is obtained if  $m_{H_u}^2$  runs to negative values at the weak scale. The relevant RGE is given by

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t \right) \quad (16)$$

with  $X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$  and where  $S = 0$  for models with universal scalars. While the gauge terms in Eq. 16 push  $m_{H_u}^2$  to larger values as  $t = \ln(Q^2)$  runs from  $m_{GUT}$  to  $m_{weak}$ , the term involving the top quark

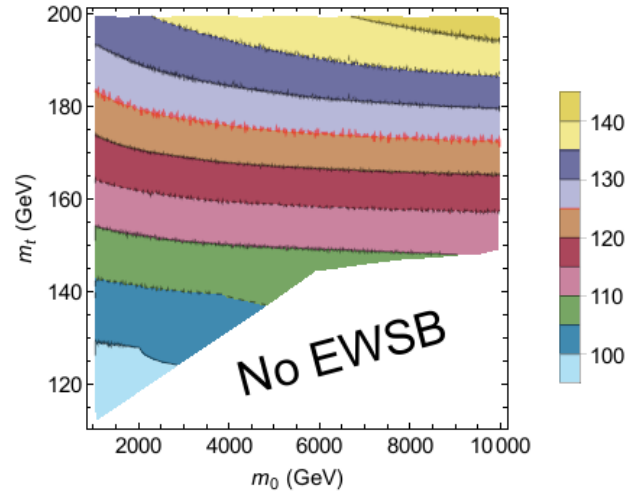


FIG. 1: Values of Higgs mass  $m_h$  which are generated for various values of  $m_t$  and  $m_0$  in a SUSY model with  $m_{1/2} = 700$  GeV and  $A_0 = -1.6m_0$  and  $\tan \beta = 10$ .

Yukawa coupling  $f_t$  pushes  $m_{H_u}^2$  towards negative values. The top-Yukawa term typically wins out for the top quark mass  $m_t \sim 100 - 200$  GeV. Had the value of  $m_t$  been found to be below  $\sim 100$  GeV, then EWSB would be hard pressed in SUSY and other exotica would have been required. The situation is shown in Fig. 1 where we show the shaded regions of the  $m_t$  vs.  $m_0$  plane where EWSB successfully occurs. For this case, we choose a mSUGRA/CMSSM model benchmark with  $m_{1/2} = 700$  GeV,  $A_0 = -1.6m_0$  and  $\tan \beta = 10$ . We also show contours of light Higgs mass  $m_h$ . In this case, a Higgs mass  $m_h \sim 125$  GeV is achieved for  $m_t \sim 175$  GeV.

#### *The mass of the Higgs boson*

In the Standard Model, the mass of the Higgs boson is given by Eq. 2. Prior to discovery, its mass could plausibly lie anywhere from the lower limit established by LEP2 searches— $m_h > 114.1$  GeV—up to  $\sqrt{8\pi\sqrt{2}/3G_F} \sim 800$  GeV as required by unitarity[29]. This mass range is exhibited in Fig. 2 as the blue band.

In contrast, in the MSSM the Higgs mass is calculated at the 1-loop level as[30]

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_W^2} \left[ \ln \frac{m_t^2}{m_t^2} + \frac{X_t^2}{m_t^2} \left( 1 - \frac{X_t^2}{12m_t^2} \right) \right] \quad (17)$$

where  $X_t = A_t - \mu \cot \beta$  and  $m_t^2 \simeq m_{Q_3} m_{U_3}$  is an effective squared stop mass. For a given  $m_t^2$ , this expression is maximal for large mixing in the top-squark sector with  $X_t^{max} = \sqrt{6}m_t$  (see Fig. 3).

For top-squark masses not much beyond the TeV scale, the upper limit on the SM-like SUSY Higgs boson is

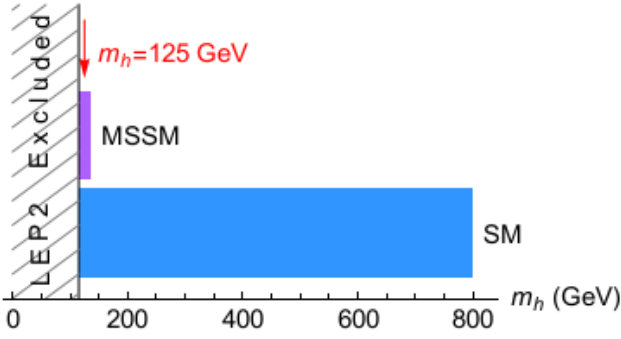


FIG. 2: Range of Higgs mass  $m_h$  predicted in the Standard Model compared to range of Higgs mass predicted by the MSSM. We also show the measured value of the Higgs mass by the arrow. The left-most region had been excluded by LEP2 searches prior to the LHC8 run.

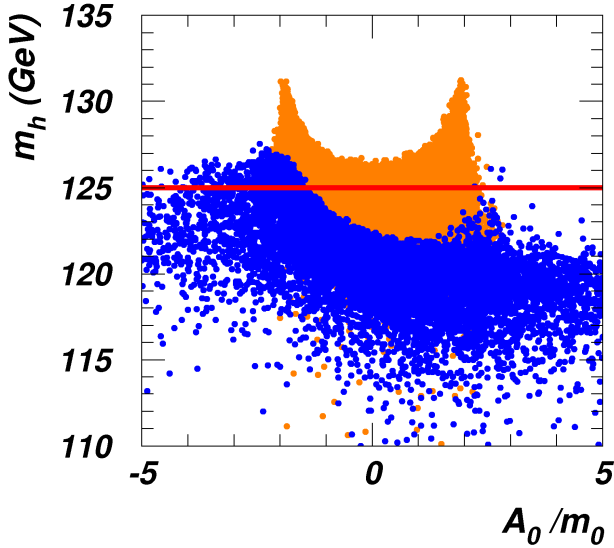


FIG. 3: Predicted mass of the light Higgs scalar  $m_h$  from a scan over NUHM2 SUSY model parameter space with  $m_0$  ranging up to 5 TeV (blue points) and up to 20 TeV (orange points) taken from Ref. [31].

$m_h \lesssim 135$  GeV. This range is shown as the purple band. The combined Atlas/CMS measured value of the newly discovered Higgs boson is given by

$$m_h = 125.15 \pm 0.24 \text{ GeV} \quad (18)$$

and is indicated by the red arrow in Fig. 2.

#### Collider searches for supersymmetric matter

While SUSY models enjoy compatibility with the measured values of the gauge couplings, the top quark mass and Higgs boson mass, the main goal is to discover supersymmetry via the direct detection of supersymmetric matter at colliding beam experiments. The CERN LEP2

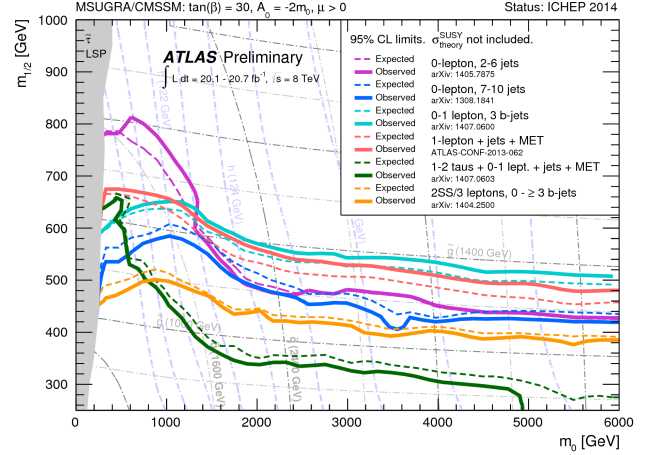


FIG. 4: Regions of the  $m_0$  vs.  $m_{1/2}$  plane which are excluded by various Atlas experiment searches for gluino and squark cascade decay signatures in the mSUGRA model with  $\tan \beta = 30$  and  $A_0 = -2m_0$ . Limits are for  $\sqrt{s} = 8$  TeV and  $20 \text{ fb}^{-1}$ .

$e^+e^-$  collider searched for SUSY in various guises without success. The most important bound to emerge from LEP2 was that chargino masses  $m_{\tilde{W}_1} > 103.5$  GeV in a relatively model-independent way as long as the mass gap  $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$  is greater than just several GeV.

At the CERN LHC, a variety of searches for SUSY particle production have taken place at  $pp$  collisions at  $\sqrt{s} = 8$  TeV. For sparticle masses in the TeV regime, the most lucrative production channel—owing to large cross sections followed by expected large energy release in cascade decays[32]—is gluino and squark pair production:  $pp \rightarrow \tilde{g}\tilde{g}$ ,  $\tilde{q}\tilde{q}$  and  $\tilde{g}\tilde{q}$ . From these processes, a variety of multi-jet plus multi-lepton plus  $E_T$  events are expected[33] provided the sparticle masses are light enough that production cross sections are sufficiently large. So far, no compelling signal has been seen above expected background levels[34, 35]. The resulting excluded regions of SUSY parameter space are shown in Fig. 4 in the context of the mSUGRA model with  $\tan \beta = 30$  and  $A_0 = -2m_0$  (values which ensure a Higgs mass  $m_h \sim 125$  GeV throughout much of the parameter space shown). The left side of the plot shows the region where squark and gluino masses are comparable  $m_{\tilde{q}} \sim m_{\tilde{g}}$  so that  $\tilde{g}\tilde{g}$ ,  $\tilde{q}\tilde{q}$  and  $\tilde{g}\tilde{q}$  can all occur at comparable rates. The right side of the plot shows the region where  $m_{\tilde{q}} \gg m_{\tilde{g}}$  so that only  $\tilde{g}\tilde{g}$  production is relevant. From the plot, we can read off the approximate bounds:

$$m_{\tilde{g}} \gtrsim 1300 \text{ GeV} \quad (m_{\tilde{g}} \ll m_{\tilde{q}}) \text{ and} \quad (19)$$

$$m_{\tilde{g}} \gtrsim 1800 \text{ GeV} \quad (m_{\tilde{g}} \sim m_{\tilde{q}}). \quad (20)$$

A vast array of further searches have taken place: for electroweak -inos, top and bottom squarks and sleptons

in mSUGRA and in simplified models and for SUSY particle production in a variety of different models. A compendium of limits can be found *e.g.* in Ref. [36].

### SUGRA gauge theories: natural or not?

While SUGRA gauge theories are both elegant and supported indirectly by data, they have come under a growing body of criticism due to a perception that they are increasingly unnatural with respect to the weak scale in light of recent LHC results on the Higgs mass and on lack of signal for sparticles (for just a few examples, see Ref's [37–45]). The increasing gap between the sparticle mass scale and the weak scale is frequently referred to as the supersymmetric Little Hierarchy Problem (LHP)[46, 47]. To see how this comes about, we must scrutinize several measures of naturalness[48–50].<sup>2</sup>

But first, an important point to be made is that *any* quantity can look fine-tuned if one splits it into *dependent* pieces. By re-writing an observable  $\mathcal{O}$  as  $\mathcal{O} + b - b$  and allowing  $b$  to be large, the quantity might look fine-tuned. In this trivial example, however, combining dependent contributions into independent units ( $b - b = 0$ ) obviously erases the presumed source of fine-tuning. To avoid such pitfalls, a simple fine-tuning rule has been proposed[50]:

When evaluating fine-tuning, it is not permissible to claim fine-tuning of *dependent* quantities one against another.

#### The electroweak measure $\Delta_{EW}$

The electroweak measure,  $\Delta_{EW}$ [87, 88], implements the Dimopoulos-Susskind requirement that there be no large/unnatural cancellations in deriving the value of  $m_Z$  from the weak scale scalar potential:

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{(\tan^2 \beta - 1)} - \mu^2 \quad (21)$$

$$\simeq -m_{H_u}^2 - \mu^2 - \Sigma_u^u \quad (22)$$

where  $m_{H_u}^2$  and  $m_{H_d}^2$  are the *weak scale* soft SUSY breaking Higgs masses,  $\mu$  is the *supersymmetric* higgsino mass term and  $\Sigma_u^u$  and  $\Sigma_d^d$  contain an assortment of loop corrections to the effective potential. The  $\Delta_{EW}$  measure compares the largest contribution on the right-hand-side of Eq. 22 to the value of  $m_Z^2/2$ . If they are comparable, then no unnatural fine-tunings are required to generate  $m_Z = 91.2$  GeV. The main requirement is then

that  $|\mu| \sim m_Z$ [91–93] (with  $\mu \gtrsim 100$  GeV to accommodate LEP2 limits from chargino pair production searches) and also that  $m_{H_u}^2$  is driven radiatively to small, and not large, negative values [87, 88].<sup>3</sup> This can always happen in models where the Higgs soft terms are non-universal, such as in the two-extra parameter non-universal Higgs model NUHM2[96]. Also, the top squark contributions to the radiative corrections  $\Sigma_u^u(\tilde{t}_{1,2})$  are minimized for TeV-scale highly mixed top squarks[87]. This latter condition also lifts the Higgs mass in Eq. 17 to  $m_h \sim 125$  GeV. The measure  $\Delta_{EW}$  is pre-programmed in the Isasugra SUSY spectrum generator[23].

One advantage of  $\Delta_{EW}$  is that— within the context of the MSSM— it is 1. *model-independent*: if a weak scale spectrum is generated within the pMSSM or via some high scale constrained model, one obtains exactly the same value of naturalness. Other virtues of  $\Delta_{EW}$  (as discussed in Ref. [88]) are that it is: 2. the most conservative of the three measures, 3. in principle measureable, 4. unambiguous, 5. predictive, 6. falsifiable and 7. simple to calculate.

The principle criticism of  $\Delta_{EW}$  is that— since it involves only weak scale parameters— it may not display the sensitivity of the weak scale to variations in high scale parameters. We will show below that the two competing measures,  $\Delta_{HS}$  and  $\Delta_{BG}$ , if implemented properly according to the fine-tuning rule, essentially reduce to  $\Delta_{EW}$  so that in fact  $\Delta_{EW}$  portrays the entirety of electroweak naturalness.

#### Large-log measure $\Delta_{HS}$

The Higgs mass fine-tuning measure,  $\Delta_{HS}$ , compares the radiative correction of the  $m_{H_u}^2$  soft term,  $\delta m_{H_u}^2$ , to the physical Higgs mass

$$m_h^2 \simeq \mu^2 + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2. \quad (23)$$

If we assume the MSSM is valid up to some high energy scale  $\Lambda$  (which may be as high as  $m_{GUT}$  or even  $m_P$ ), then the value of  $\delta m_{H_u}^2$  can be found by integrating the renormalization group equation (RGE):

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2} \left( -\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t \right) \quad (24)$$

where  $t = \ln(Q^2/Q_0^2)$ ,  $S = m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2]$  and  $X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$ . By neglecting gauge terms and  $S$  ( $S = 0$  in models with scalar soft term universality but can be

<sup>2</sup> Further investigations into naturalness in SUSY theories include Ref's [48, 51–90].

<sup>3</sup> Some recent work on theories with naturalness and heavy higgsinos include [94, 95].

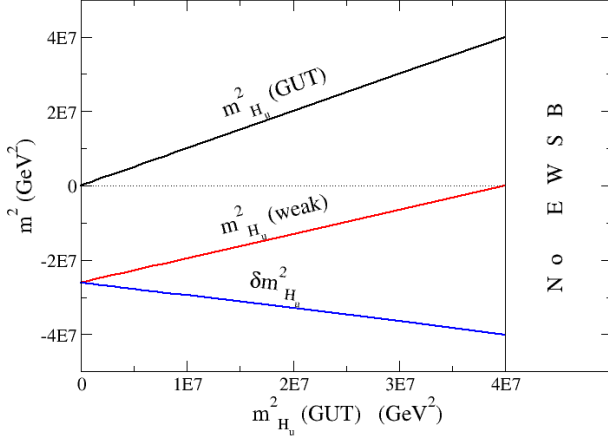


FIG. 5: Values of  $\delta m^2_{H_u}$ ,  $m^2_{H_u}(m_{GUT})$  and  $m^2_{H_u}(m_{weak})$  vs.  $m^2_{H_u}(m_{GUT})$  for a model with  $m_0 = 5$  TeV,  $m_{1/2} = 700$  GeV and  $A_0 = -8$  TeV and  $\tan \beta = 10$ . Here,  $m^2_{H_u}(weak) = m^2_{H_u}(m_{GUT}) + \delta m^2_{H_u}$ .

large in models with non-universality), and also neglecting the  $m^2_{H_u}$  contribution to  $X_t$  and the fact that  $f_t$  and the soft terms evolve under  $Q^2$  variation, then a simple expression may be obtained by integrating from  $m_{SUSY}$  to the cutoff  $\Lambda$ :

$$\delta m^2_{H_u} \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{SUSY}^2). \quad (25)$$

Here, we take as usual  $m_{SUSY}^2 \simeq m_{\tilde{t}_1} m_{\tilde{t}_2}$ . By requiring[97–100]

$$\Delta_{HS} \sim \delta m^2_{H_u}/(m_h^2/2) \lesssim 10 \quad (26)$$

then one expects the three third generation squark masses  $m_{\tilde{t}_{1,2}, \tilde{b}_1} \lesssim 600$  GeV. Using the  $\Delta_{HS}$  measure of fine-tuning along with  $m_h \simeq 125$  GeV, one finds some popular SUSY models to be electroweak fine-tuned to 0.1%[48].

Two pitfalls occur within this approach, which are *different* from the case of the SM.

- The first is that  $m^2_{H_u}(\Lambda)$  and  $\delta m^2_{H_u}$  are *not* independent: the value of  $m^2_{H_u}$  feeds directly into evaluation of  $\delta m^2_{H_u}$  via the  $X_t$  term. It also feeds indirectly into  $\delta m^2_{H_u}$  by contributing to the evolution of the  $m_{Q_3}^2$  and  $m_{U_3}^2$  terms. This can be seen in Fig. 5 where we plot  $\delta m^2_{H_u}$  as a function of  $m^2_{H_u}(m_{GUT})$  for a particular choice of model parameters. We see that as  $m^2_{H_u}(m_{GUT})$  increases, then there is an increasingly *negative* cancelling correction  $\delta m^2_{H_u}$ . Thus, this measure *fails* the fine-tuning rule[50].

- A second issue with  $\Delta_{HS}$  is that— whereas  $SU(2)_L \times U(1)_Y$  gauge symmetry can be broken at tree level in the SM— in the SUGRA case, where SUSY is broken in a hidden sector via the super-Higgs mechanism, then  $m^2_{H_u} \sim m_{3/2}^2 > 0$ . Thus, for SUGRA models, electroweak symmetry is not even broken until one includes radiative corrections. For SUSY models valid up to some high scale  $\Lambda \gg m_{weak}$ , the large log in Eq. 25 is exactly what is required to break EW symmetry in the first place, radiatively driving  $m^2_{H_u}$  to negative values[25].

A simple fix for  $\Delta_{HS}$  is to *combine the dependent terms* into a single quantity. Under such a regrouping[87, 88], then

$$m_h^2|_{phys} = \mu^2 + (m^2_{H_u}(\Lambda) + \delta m^2_{H_u}) \quad (27)$$

where now  $\mu^2$  and  $(m^2_{H_u}(\Lambda) + \delta m^2_{H_u})$  are each independent so each should be comparable to  $m_h^2$  in order to avoid fine-tuning. The large log is still present in  $(m^2_{H_u}(\Lambda) + \delta m^2_{H_u})$ , but now cancellations can occur between the boundary condition and the radiative correction.

It is sometimes claimed that under such a regrouping, then the SM Higgs mass would also not be fine-tuned. But here we see that, in the MSSM case— since the  $m^2_{H_u}$  and  $\delta m^2_{H_u}$  terms are dependent— the situation is different from the SM and one must combine dependent terms together. The regrouping of contributions to  $m_h^2$  leads back to the  $\Delta_{EW}$  measure since now  $(m^2_{H_u}(\Lambda) + \delta m^2_{H_u}) = m^2_{H_u}(weak)$ . Indeed, we see from Fig. 5 that for large enough  $m^2_{H_u}(m_{GUT})$ , then there can be a large cancellation so that the weak scale value of  $|m^2_{H_u}|$  is indeed comparable to  $m_Z^2$ . This is the case of radiatively-driven naturalness.

#### Sensitivity to high scale parameters: $\Delta_{BG}$

The more traditional measure,  $\Delta_{BG}$ , was proposed by Ellis *et al.*[101] and later investigated more thoroughly by Barbieri and Giudice[102]. The starting point is to express  $m_Z^2$  in terms of weak scale SUSY parameters as in Eq. 22:

$$m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2, \quad (28)$$

where the partial equality obtains for moderate-to-large  $\tan \beta$  values and where we assume for now the radiative corrections are small. To evaluate  $\Delta_{BG}$ , one needs to know the explicit dependence of the weak scale values of  $m_{H_u}^2$  and  $\mu^2$  on the fundamental parameters. Semi-analytic solutions to the one-loop renormalization group equations for  $m_{H_u}^2$  and  $\mu^2$  can be found for instance in



Ref's [103]. For the case of  $\tan \beta = 10$ , then[89, 104, 105]

$$\begin{aligned}
m_Z^2 = & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 \\
& + 0.047M_1M_3 - 0.42M_2^2 + 0.011M_2M_1 - 0.012M_1^2 \\
& - 0.65M_3A_t - 0.15M_2A_t - 0.025M_1A_t + 0.22A_t^2 \\
& + 0.004M_3A_b - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
& + .73m_{Q_3}^2 + .57m_{U_3}^2 + .049m_{D_3}^2 - .052m_{L_3}^2 + .053m_{E_3}^2 \\
& + .051m_{Q_2}^2 - .11m_{U_2}^2 + .051m_{D_2}^2 - .052m_{L_2}^2 + .053m_{E_2}^2 \\
& + .051m_{Q_1}^2 - .11m_{U_1}^2 + .051m_{D_1}^2 - .052m_{L_1}^2 + .053m_{E_1}^2,
\end{aligned} \tag{29}$$

where the parameters on the right-hand-side are understood as evaluated at the GUT scale. (For different values of  $\tan \beta$ , then somewhat different co-efficients are obtained.)

Then, the proposal is that the variation in  $m_Z^2$  with respect to parameter variation be small:

$$\Delta_{BG} \equiv \max_i [c_i] \quad \text{where} \quad c_i = \left| \frac{\partial \ln m_Z^2}{\partial \ln p_i} \right| = \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right| \tag{30}$$

where the  $p_i$  constitute the fundamental parameters of the model. Thus,  $\Delta_{BG}$  measures the fractional change in  $m_Z^2$  due to fractional variation in high scale parameters  $p_i$ . The  $c_i$  are known as *sensitivity coefficients*[102].

The requirement of low  $\Delta_{BG}$  is then equivalent to the requirement of no large cancellations on the right-hand-side of Eq. 29 since (for linear terms) the logarithmic derivative just picks off coefficients of the relevant parameter. For instance,  $c_{m_{Q_3}^2} = 0.73 \cdot (m_{Q_3}^2/m_Z^2)$ . If one allows  $m_{Q_3} \sim 3$  TeV (in accord with requirements from the measured value of  $m_h$ ), then one obtains  $c_{m_{Q_3}^2} \sim 800$  and so  $\Delta_{BG} \geq 800$ . In this case, SUSY would be electroweak fine-tuned to about 0.1%. If instead one insists that  $m_0$  is the fundamental parameter with  $m_{Q_3} = m_{U_3} = m_{H_u} \equiv m_0$ , as in models with scalar mass universality, then the various scalar mass contributions to  $m_Z^2$  largely *cancel* and  $c_{m_0^2} \sim -0.017m_0^2/m_Z^2$ : the contribution to  $\Delta_{BG}$  from scalars drops by a factor  $\sim 50$ [89].

The above example illustrates the extreme model-dependence of  $\Delta_{BG}$  for multi-parameter SUSY models. The value of  $\Delta_{BG}$  can change radically from theory to theory even if those theories generate exactly the same weak scale sparticle mass spectrum. The model dependence of  $\Delta_{BG}$  arises due to a violation of the Fine-tuning Rule: one must combine dependent terms into independent quantities before evaluating EW fine-tuning.

#### $\Delta_{BG}$ applied to SUGRA gauge theories

In Ref. [48], it was argued that: in an ultimate theory (UTH), where all soft parameters are correlated, then

$\Delta_{BG}$  should be a reliable measure of naturalness. In fact, SUGRA gauge theories with hidden sector SUSY breaking fulfill this requirement. The amazing thing is that we do not need to know the precise hidden sector in order to properly evaluate  $\Delta_{BG}$ .

In supergravity gauge theories with hidden sector SUSY breaking via the superHiggs mechanism, where the hidden sector is fully specified, the gravitino gains a mass  $m_{3/2}$  but then in addition all soft SUSY breaking terms are generated as multiples of the gravitino mass  $m_{3/2}$ . Thus, we can write each soft term as

$$m_{H_u}^2 = a_{H_u} \cdot m_{3/2}^2, \tag{31}$$

$$m_{Q_3}^2 = a_{Q_3} \cdot m_{3/2}^2, \tag{32}$$

$$A_t = a_{A_t} \cdot m_{3/2}, \tag{33}$$

$$M_i = a_i \cdot m_{3/2}, \tag{34}$$

$$\dots \tag{35}$$

For any fully specified hidden sector, the various  $a_i$  are calculable. For example, in string theory with dilaton-dominated SUSY breaking[14, 15], we expect  $m_0^2 = m_{3/2}^2$  with  $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$ . Alternatively, acknowledging our lack of knowledge of hidden sector dynamics, we may *parametrize our ignorance* by leaving the  $a_i$  as free parameters. By using several adjustable parameters, we cast a wide net which encompasses a large range of hidden sector SUSY breaking possibilities. But this doesn't mean that each SSB parameter is expected to be independent of the others. It just means we do not know how SUSY breaking occurs, and how the soft terms are correlated: it is important not to confuse parameters, which ought to be related to one another in any sensible theory of SUSY breaking, with independently adjustable soft SUSY breaking terms.

Now, plugging the soft terms Eq's 31-35 into Eq. 29, one arrives at the simpler expression

$$m_Z^2 = -2.18\mu^2 + a \cdot m_{3/2}^2. \tag{36}$$

The value of  $a$  is just some number which is the sum of all the coefficients of the terms  $\propto m_{3/2}^2$ . For now, we assume  $\mu$  is independent of  $m_{3/2}$  as will be discussed below.

Using Eq. 36, we can compute the sensitivity coefficients in the theory where the soft terms are properly correlated:<sup>4</sup>

$$c_{m_{3/2}^2} = |a \cdot (m_{3/2}^2/m_Z^2)| \quad \text{and} \tag{37}$$

$$c_{\mu^2} = |-2.18(\mu^2/m_Z^2)|. \tag{38}$$

<sup>4</sup> In mAMSB, the soft terms are also written as multiples of  $m_{3/2}$  or  $m_{3/2}^2$ . In mGMSB, the soft terms are written as multiples of messenger scale  $\Lambda_m$ . The argument proceeds in an identical fashion in these cases.



For  $\Delta_{BG}$  to be  $\sim 1 - 10$  (natural SUSY with low fine-tuning), then Eq. 38 implies

- $\mu^2 \sim m_Z^2$ .

Also, Eq. 37 implies

- $a \cdot m_{3/2}^2 \sim m_Z^2$ .

The first of these conditions implies light higgsinos with mass  $\sim 100 - 200$  GeV, the closer to  $m_Z$  the better. The second condition can be satisfied if  $m_{3/2} \sim m_Z$  [102] (which now seems highly unlikely due to a lack of LHC8 SUSY signal<sup>5</sup> and the rather large value of  $m_h$ ) or if  $a$  is quite small: in this latter case, the SSB terms conspire such that there are large cancellations amongst the various coefficients of  $m_{3/2}^2$  in Eq. 29: this is what is called radiatively-driven natural SUSY [87, 88] since in this case a large high scale value of  $m_{H_u}^2$  can be driven radiatively to small values  $\sim -m_Z^2$  at the weak scale.

Furthermore, we can equate the value of  $m_Z^2$  in terms of weak scale parameters with the value of  $m_Z^2$  in terms of GUT scale parameters:

$$\begin{aligned} m_Z^2 &\simeq -2\mu^2(weak) - 2m_{H_u}^2(weak) \\ &\simeq -2.18\mu^2(GUT) + a \cdot m_{3/2}^2. \end{aligned} \quad (39)$$

Since  $\mu$  hardly evolves under RG running (the factor 2.18 is nearly 2), then we have the BG condition for low fine-tuning as

$$-2m_{H_u}^2(weak) \sim a \cdot m_{3/2}^2 \sim m_Z^2, \quad (40)$$

*i.e.* that the value of  $m_{H_u}^2$  must be driven to *small* negative values  $\sim -m_Z^2$  at the weak scale. These are exactly the conditions required by the model-independent EWFT measure  $\Delta_{EW}$ : *i.e.* we have

$$\lim_{n_{SSB} \rightarrow 1} \Delta_{BG} \rightarrow \Delta_{EW} \quad (41)$$

where  $n_{SSB}$  is the number of *independent* soft SUSY breaking terms. In this sense, a low value of  $\Delta_{EW}$  reflects not only low weak scale fine-tuning, but also low high scale fine-tuning! Of course, this approach also reconciles the Higgs mass fine-tuning measure  $\Delta_{HS}$  (with appropriately regrouped independent terms) with the  $\Delta_{BG}$  measure (when applied to models with a single independent soft breaking term such as  $m_{3/2}$ ).

*A worked example: BG and EW fine-tuning in a model with a Polonyi-type hidden sector*

As a concrete example, we evaluate  $\Delta_{BG}$  in a model with  $m_{\tilde{q}} = m_{\tilde{t}} = m_{H_{u,d}} = 1$  TeV,  $A_0 = 1268$  GeV

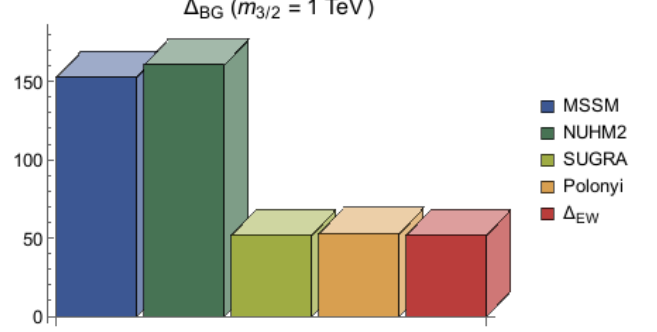


FIG. 6: Values of  $\Delta_{BG}$  computed in a model with  $m_{\tilde{q}} = m_{\tilde{t}} = m_{H_{u,d}} = 1$  TeV,  $A_0 = 1268$  GeV and  $\tan \beta = 10$  with  $m_{1/2} = m_{\tilde{q}}/3$ . We show  $\Delta_{BG}$  as computed for the MSSM model, the NUHM2 model, the mSUGRA model and the Polonyi model with  $m_{3/2} = 1$  TeV. We also show the value of  $\Delta_{EW}$ .

and  $\tan \beta = 10$  and with  $m_{1/2} = m_{\tilde{q}}/3$ . In the MSSM, the largest sensitivity co-efficient comes from the  $m_{H_u}^2$  term in Eq. 29 yielding  $\Delta_{BG} = \Delta_{H_u} = 1.27m_{H_u}^2(m_{GUT})/m_Z^2 = 153$ . If we notice the squark and slepton masses are universal, we may instead evaluate in the NUHM2 model where the combined 3rd generation terms in Eq. 29 give the largest contribution:  $\Delta_{BG} = \Delta_{m_{0(3)}} = 1.35m_0^2/m_Z^2 = 162$ . If we further notice that the Higgs and matter scalar soft terms are degenerate at  $m_{GUT}$  ( $m_{H_u}^2 = m_{H_d}^2 = m_0^2$ ), we may instead evaluate in the mSUGRA/CMSSM model. In this case, the combined third generation and Higgs contributions to Eq. 29 largely cancel so that the largest soft term contribution comes from  $\Delta_{m_{1/2}} = m_{1/2}(7.572m_{1/2} - 0.821A_0)/(2m_Z^2) = 29.6$ . However,  $\mu^2$  should really be regarded an input parameter where  $\mu(gut) = 445$  GeV. In this case,  $\Delta_{BG} = \Delta_{\mu^2} = 2.18\mu^2(gut)/m_Z^2 = 51.9$ . If we make a final realization that the soft terms are exactly those of the Polonyi model Eq. 7— all computed as multiples of  $m_{3/2}$  with  $m_{1/2} = m_{3/2}/3$ — then  $\Delta_{BG} = \Delta_{m_{3/2}} = 0.44m_{3/2}^2/m_Z^2 = 52.9$ . These values are displayed in the histograms of Fig. 6. We also compare against the value of  $\Delta_{EW} \simeq \mu^2(weak)/(m_Z^2/2) = 51.9$  since  $\mu(weak) = 465$  GeV in order to enforce  $m_Z = 91.2$  GeV. Thus, we see that  $\Delta_{EW}$  is a good approximation to  $\Delta_{BG}$  when evaluated using Eq. 36 where  $a$  turns out to be 0.44.

### Radiatively-driven naturalness

We have seen that, when applied appropriately, the three measures of SUSY weak scale naturalness are in accord:

$$\Delta_{EW} \simeq \Delta_{HS} \simeq \Delta_{BG}. \quad (42)$$

<sup>5</sup> For instance, in simple SUGRA models, the scalar masses  $m_0 = m_{3/2}$ . Since LHC requires rather high  $m_0$ , then we would also expect rather large  $m_{3/2}$ .

Thus, in the following discussion we will use the EW measure due to the ease of including radiative corrections: the 43 terms of  $\Sigma_u^u$  and  $\Sigma_d^d$  which are listed in the Appendix of Ref. [88]. The requirements for natural SUSY are then plain to see:

- the soft term  $m_{H_u}^2$  is driven radiatively to small negative values  $\sim -m_Z^2$  at the weak scale,
- the  $\mu$  parameter independently is of magnitude  $\sim m_Z$ , the closer to  $m_Z$  the better, and
- the radiative corrections  $|\Sigma_u^u|$  should be not much larger than  $m_Z^2$ . The largest contribution to  $\Sigma_u^u$  comes almost always from the stop sector. Using the exact one-loop radiative corrections, the large  $A_t$  trilinear soft term suppresses both the terms  $\Sigma_u^u(\tilde{t}_{1,2})$  whilst lifting  $m_h$  to  $\sim 125$  GeV[87]. Since under RG running the gluino soft term  $M_3$  lifts the stop masses, a limit on the contribution  $\Sigma_u^u(\tilde{t}_{1,2})$  also provides a (two-loop) upper bound on  $m_{\tilde{g}}$ .

The term  $m_{H_d}^2$  is suppressed by  $\tan^2 \beta$  in Eq. 22 and so can be much larger: in the multi-TeV range without violating naturalness. Since the heavy SUSY Higgs masses  $m_{A,H,H^\pm} \sim m_{H_d}$ , then these could all live in the TeV range, perhaps beyond the reach of LHC13[106].

A scan over NUHM2 parameter space yields the plot of  $m_{\tilde{g}}$  vs.  $\Delta_{EW}$  in Fig. 7. Here, we see that for  $\Delta_{EW} < 30$ , the upper bound on  $m_{\tilde{g}}$  extends to  $\sim 4$  TeV, well beyond the ultimate reach of LHC. If instead we require  $\Delta_{EW} \lesssim 10$ , then  $m_{\tilde{g}} \lesssim 2$  TeV, and should be accessible to LHC searches. In Fig. 8, we show the value of  $m_{\tilde{t}_1}$  vs.  $\Delta_{EW}$ . Here, we see that light stop masses can exist in the 1-2 TeV range while maintaining naturalness. In Fig. 9, we show the mass of the lightest charged higgsino  $m_{\tilde{W}_1}$  vs.  $\Delta_{EW}$ . For  $\Delta_{EW} \lesssim 30$ , then  $m_{\tilde{W}_1} \lesssim 300$  GeV so that a linear  $e^+e^-$  collider operating with  $\sqrt{s} \sim 600$  GeV will probe the entire space with modest values of  $\Delta_{EW}$ .

A typical sparticle mass spectrum with radiatively-driven naturalness is shown in Fig. 10.

### QCD naturalness, Peccei-Quinn symmetry and the $\mu$ problem

While on the topic of naturalness, we should include discussion of naturalness in the QCD sector. In the early days of QCD, it was a mystery why the two-light-quark chiral symmetry  $U(2)_L \times U(2)_R$  gave rise to three and not four light pions[107]. The mystery was resolved by 't Hooft's discovery of the QCD theta vacuum which didn't respect the  $U(1)_A$  symmetry[108]. As a consequence of the theta vacuum, one expects the presence of a term

$$\mathcal{L} \ni \frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu} \tilde{F}_A^{\mu\nu} \quad (43)$$

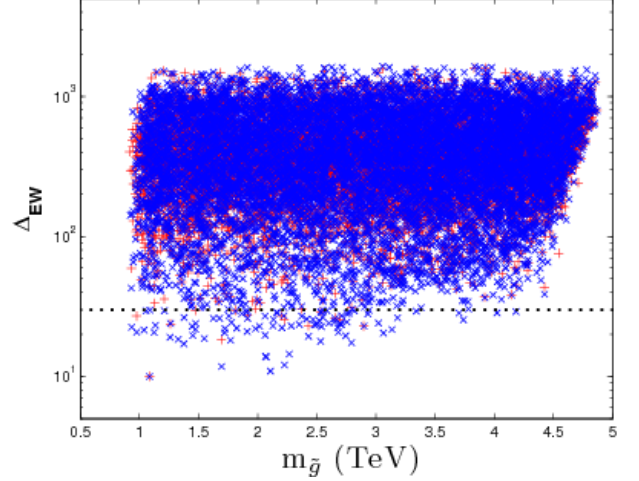


FIG. 7: Plot of  $m_{\tilde{g}}$  vs.  $\Delta_{EW}$  from a scan over NUHM2 parameter space (Ref. [88]). Blue points represent a focussed scan at low  $\mu$  while red points represent a scan over a broader range of  $\mu$ .

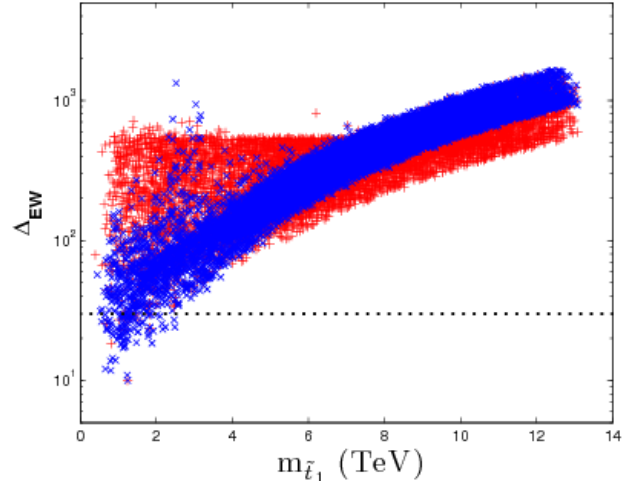


FIG. 8: Plot of  $m_{\tilde{t}_1}$  vs.  $\Delta_{EW}$  from a scan over NUHM2 parameter space (Ref. [88]). Blue points represent a focussed scan at low  $\mu$  while red points represent a scan over a broader range of  $\mu$ .

in the QCD Lagrangian (where  $\bar{\theta} = \theta + \arg(\det(\mathcal{M}))$  and  $\mathcal{M}$  is the quark mass matrix). Measurements of the neutron EDM constrain  $\bar{\theta} \lesssim 10^{-10}$  leading to an enormous fine-tuning in  $\theta$ : the so-called strong CP problem.

The strong CP problem is elegantly solved via the PQWW[109] introduction of PQ symmetry and the concomitant (invisible[110, 111]) axion: the offending term can dynamically settle to zero. The axion is a valid dark matter candidate in its own right[112].

Introducing the axion in a SUSY context solves the strong CP problem but also offers an elegant solution

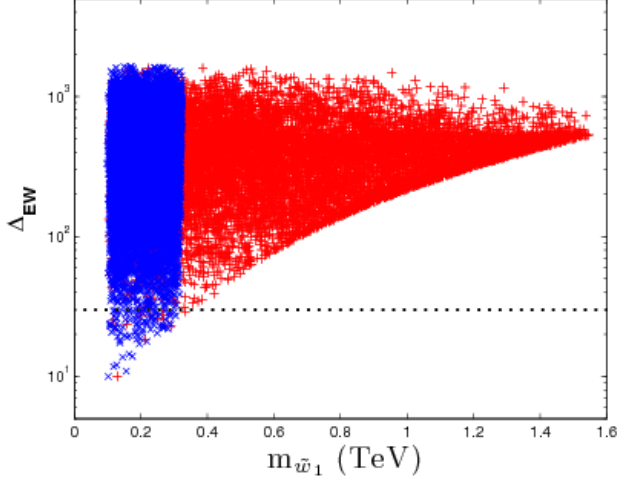


FIG. 9: Plot of  $m_{\tilde{W}_1}$  vs.  $\Delta_{EW}$  from a scan over NUHM2 parameter space (Ref. [88]). Blue points represent a focussed scan at low  $\mu$  while red points represent a scan over a broader range of  $\mu$ .

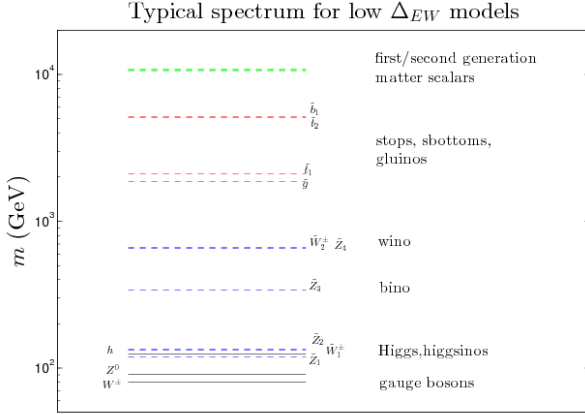


FIG. 10: Typical sparticle mass spectrum from SUSY models with low  $\Delta_{EW}$ , *i.e.* radiatively-driven naturalness.

to the SUSY  $\mu$  problem. The most parsimonious implementation of the strong CP solution involves introducing a single MSSM singlet superfield  $S$  carrying PQ charge  $Q_{PQ} = -1$  while the Higgs fields both carry  $Q_{PQ} = +1$ . The usual  $\mu$  term is forbidden, but then we have a superpotential[113]

$$W_{DFSZ} \ni \lambda \frac{S^2}{M_P} H_u H_d. \quad (44)$$

If PQ symmetry is broken and  $S$  receives a VEV  $\langle S \rangle \sim f_a$ , then a weak scale  $\mu$  term

$$\mu \sim \lambda f_a^2 / M_P \quad (45)$$

is induced which gives  $\mu \sim m_Z$  for  $f_a \sim 10^{10}$  GeV. While Kim-Nilles sought to relate the PQ breaking scale  $f_a$  to

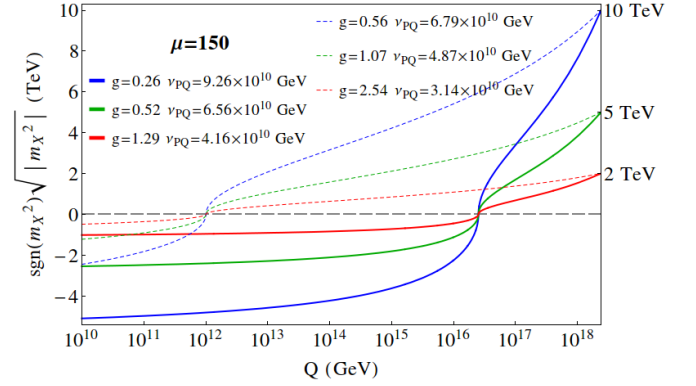


FIG. 11: Plot of the running values of  $m_X^2$  versus  $Q$  for various values of  $m_{3/2}$  and  $h = 2$  (dashed) and  $h = 4$  (solid). Here,  $g$  and  $h$  are couplings from the MSY model Lagrangian[47, 114].

the hidden sector mass scale  $m_{hidden}$ [28], we see now that the Little Hierarchy

$$\mu \sim m_Z \ll m_{3/2} \sim \text{multi-TeV} \quad (46)$$

could emerge due to a mis-match between PQ breaking scale and hidden sector mass scale  $f_a \ll m_{hidden}$ .

In fact, an elegant model which exhibits this behavior was put forth by Murayama, Sakai and Yanagida (MSY)[114]. In the MSY model, PQ symmetry is broken radiatively by driving one of the PQ scalars  $X$  to negative mass-squared values in much the same way that electroweak symmetry is broken by radiative corrections driving  $m_{H_u}^2$  negative. Starting with multi-TeV scalar masses, the radiatively-broken PQ symmetry induces a SUSY  $\mu$  term  $\sim 100$  GeV[47] while at the same time generating intermediate scale Majorana masses for right-hand neutrinos: see Fig. 11. In models such as MSY, the Little Hierarchy  $\mu \ll m_{3/2}$  is no problem at all but is instead just a reflection of the mis-match between PQ and hidden sector mass scales.

## IMPLICATIONS

### LHC searches for SUSY with radiatively-driven naturalness

SUSY models with radiatively-driven naturalness have been examined in the case of gaugino mass unification (where the LSP is a higgsino-like neutralino) and in lesser detail for the case of non-unified gaugino masses (where the LSP could be either bino-like or wino-like while preserving naturalness)[115]. Here, we briefly summarize prospects for the more motivated case with gaugino mass unification.

For sparticle searches at LHC13, the best prospects for the next couple years will be in searches for gluino and

Int. lum. (fb <sup>-1</sup> )	$\tilde{g}\tilde{g}$	SSdB	$WZ \rightarrow 3\ell$	$4\ell$
10	1.4	—	—	—
100	1.6	1.6	—	$\sim 1.2$
300	1.7	2.1	1.4	$\gtrsim 1.4$
1000	1.9	2.4	1.6	$\gtrsim 1.6$

TABLE I: Reach of LHC14 for SUSY in terms of gluino mass,  $m_{\tilde{g}}$  (TeV), assuming various integrated luminosity values along the RNS model line. We present each search channel considered in this paper except soft  $3\ell$ .

squark pair production. While squark masses can range into the 10-20 TeV range while not compromising naturalness<sup>6</sup>, gluino masses are required to be below about 2 TeV for  $\Delta_{EW} \lesssim 10$  and less than about 5 TeV for  $\Delta_{EW} \lesssim 30$ [88]. Thus, a lucrative portion of RNS parameter space will be accessible via gluino pair searches at LHC13. In searching for  $pp \rightarrow \tilde{g}\tilde{g}X$  production, in RNS models the dominant gluino decay is to third generation quarks:  $\tilde{g} \rightarrow t\tilde{t}_1$  (followed by  $\tilde{t} \rightarrow b\tilde{W}_i$ ) if kinematically allowed or to three-body modes  $t\tilde{t}\tilde{Z}_i$  or  $tb\tilde{W}_i$  if two-body modes are closed[117]. These decays will yield the usual multi-jet + multi-isolated-lepton +  $E_T$  gluino cascade decay events albeit ones that are rich in identifiable  $b$ -jets[33, 118]. The mass edge at  $m(\ell^+\ell^-) < m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$  arising from  $\tilde{Z}_2 \rightarrow \tilde{Z}_1\ell\bar{\ell}$  decay[33] may be apparent in cascade decay events containing OS/SF dileptons. The LHC reach for RNS is shown in Table I (for  $\sqrt{s} = 14$  TeV) in terms of  $m_{\tilde{g}}$  where squarks are assumed very heavy[119]. The LHC reach for gluino pair production cascade decay signatures extends to  $m_{\tilde{g}} \sim 1.9$  TeV for 1000 fb<sup>-1</sup> of integrated luminosity.

Since the higgsino states  $\tilde{W}_1^\pm$  and  $\tilde{Z}_{1,2}$  are so light in RNS, they tend to provide the dominant SUSY production cross section. However, the heavier higgsino states decay via three-body mode to lighter higgsino states:  $\tilde{W}_1 \rightarrow f\bar{f}'\tilde{Z}_1$  and  $\tilde{Z}_2 \rightarrow \tilde{Z}_1f\bar{f}$ . Since the inter-higgsino mass gap is so small—typically just 10-20 GeV—there is very little visible energy release as most of the energy goes into making up the LSP mass  $m_{\tilde{Z}_1}$  which serves as (a portion of) the dark matter. Thus, the higgsino pair production reactions seem very difficult to see at LHC above SM processes. It is possible that making use of initial state jet radiation may help marginally in extracting a signal for light higgsino pair production[120–123].

For SUSY models with light higgsinos, a very dis-

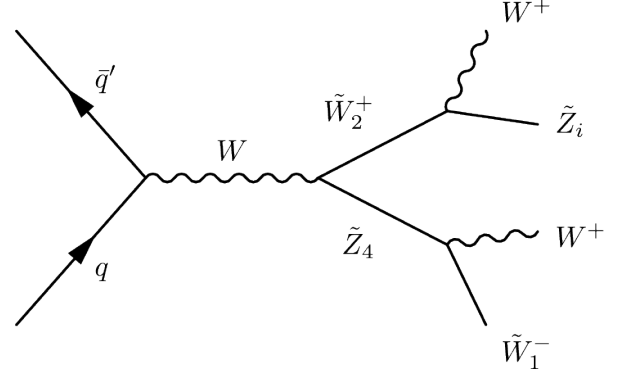


FIG. 12: Diagram depicting same-sign diboson production at LHC in SUSY models with light higgsinos.

tinctive, and ultimately more powerful, search channel emerges: that of same-sign diboson production (SSdB)[124] as shown in Fig. 12. In RNS models with gaugino mass unification, the  $\tilde{W}_2$  and  $\tilde{Z}_4$  are wino-like and tend to provide the largest *visible* SUSY cross section over the expected range of  $m_{\tilde{g}}$ . This is simply because  $\sigma(\tilde{g}\tilde{g})$  is rapidly decreasing with increasing  $m_{\tilde{g}}$  and so pair production of the lighter wino-pairs  $pp \rightarrow \tilde{W}_2^\pm \tilde{Z}_4$  wins out. The dominant wino decay modes include  $\tilde{W}_2^\pm \rightarrow \tilde{Z}_{1,2}W^\pm$  and  $\tilde{Z}_4 \rightarrow \tilde{W}_1^\pm W^\mp$ . As mentioned above, the higgsino states  $\tilde{W}_1$  and  $\tilde{Z}_{1,2}$  yield only soft decay products and are quasi-invisible. The final state then consists of same-sign or opposite-sign dibosons +  $E_T$ . While the OS diboson signal is expected to be buried under a prodigious SM  $W^+W^-$  background, the background for the same-sign diboson topology is very low. A detailed signal/background study in Ref's [119, 124] find the SSdB channel to ultimately give the best reach of LHC13 for SUSY. In Table I, for 1000 fb<sup>-1</sup> the LHC14 reach via the SSdB channel extends to  $m_{\tilde{g}} \sim 2.4$  TeV (compared against 1.9 TeV for the reach via  $\tilde{g}\tilde{g}$  cascade decays). While the SSdB channel gives the maximal LHC reach for RNS, it is also important to note that this channel is distinctive to models with light higgsinos and would provide strong confirmation for natural SUSY.

### The ILC: a higgsino factory

The smoking-gun signature of SUSY with radiatively-driven naturalness is the presence of four light higgsino states  $\tilde{Z}_{1,2}$  and  $\tilde{W}_1^\pm$  with mass  $\sim |\mu|$  and where  $|\mu| \sim 100 - 200$  GeV. Thus, these states should be accessible to a linear  $e^+e^-$  collider operating with  $\sqrt{s} > 2m(\text{higgsino})$ . While the 10-20 GeV inter-higgsino mass gaps are problematic at LHC, they should be easily visible in the clean environment of an  $e^+e^-$  collider.

Fig. 13 shows various RNS SUSY cross sections versus  $\sqrt{s}$  at the ILC. The important point is that while

<sup>6</sup> Such heavy matter scalars provide a decoupling solution[116] to the SUSY flavor and  $CP$  problems and are favored by a heavy gravitino solution to the gravitino problem. However, such heavy squarks/sleptons can lead to large loop-level contributions to  $m_Z$  unless certain GUT relations amongst masses are obeyed[90].

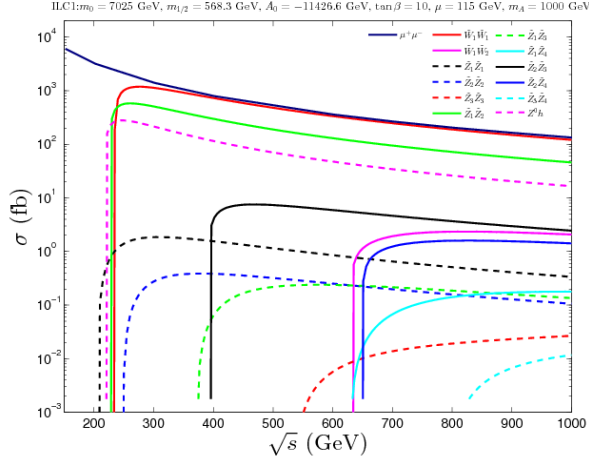


FIG. 13: Sparticle production cross sections vs.  $\sqrt{s}$  for unpolarized beams at the ILC  $e^+e^-$  collider for an RNS benchmark point listed in Ref. [125].

one expects ILC to be constructed as a Higgs factory ( $e^+e^- \rightarrow Zh$ ), it stands an excellent chance to emerge as a SUSY discovery machine and a *higgsino factory*! (The limited beam energy ( $\sqrt{s} \sim 350$  GeV) of a machine like TLEP may or may not be sufficient to produce the required light higgsino pairs.) From Fig. 13, we see that the dominant higgsino pair production reactions would consist of  $e^+e^- \rightarrow \tilde{W}_1^+ \tilde{W}_1^-$  and  $\tilde{Z}_1 \tilde{Z}_2$ . Detailed studies of signal and background[125] find that the higgsino pair production reactions should be straightforward to extract from SM background including  $\gamma\gamma$ -initiated events. Further, making use of the  $\tilde{W}_1 \rightarrow q\bar{q}' \tilde{Z}_1$  and  $\tilde{W}_1 \rightarrow \ell\bar{\nu} \tilde{Z}_1$  events, the  $\tilde{W}_1$  and  $\tilde{Z}_1$  masses can be extracted. Also, the  $\tilde{Z}_2$  and  $\tilde{Z}_1$  masses can be extracted from  $\tilde{Z}_1 \tilde{Z}_2$  production followed by  $\tilde{Z}_2 \rightarrow \ell^+ \ell^- \tilde{Z}_1$  decay. The higgsino-like nature of the particles is easily extracted using event kinematics and beam polarization. As  $\sqrt{s}$  is increased, further SUSY pair production reactions should successively be accessed.

### Dark matter: an axion/WIMP admixture?

As mentioned above, to allow for both electroweak and QCD naturalness, one needs a model including both axions and SUSY. In such a case, the axion field is promoted to a superfield which contains a spin-0  $R$ -parity even saxion  $s$  and a spin-1/2  $R$ -parity odd axino  $\tilde{a}$ . Typically in SUGRA one expects the saxion mass  $m_s \sim m_{3/2}$  and the axino mass  $m_{\tilde{a}} \lesssim m_{3/2}$ . The dark matter is then comprised of two particles: the axion along with the LSP which is a Higgsino-like WIMP. This is good news for natural SUSY since thermal higgsino-like WIMPs are typically underproduced by a factor 10-15 below the measured dark matter abundance. The remainder can be

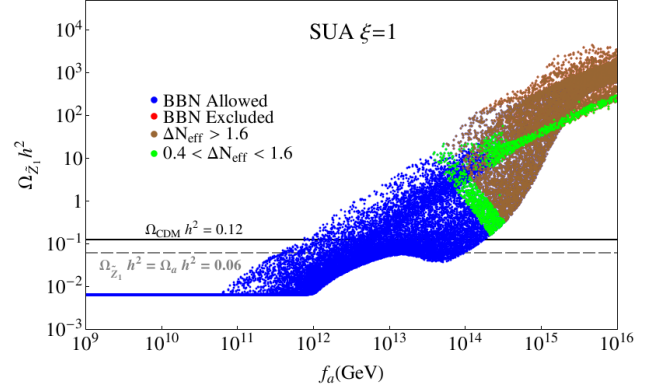


FIG. 14: The neutralino relic density from a scan over SUSY DFSZ parameter space for the RNS benchmark case labelled SUA with  $\xi = 1$ . The grey dashed line shows the points where DM consists of 50% axions and 50% neutralinos. The red BBN-forbidden points occur at  $f_a \sim 10^{14}$  GeV and are covered over by the brown  $\Delta N_{eff} > 1.6$  coloration. This latter region is excluded by Planck limits[130] of dark radiation as parametrized by additional neutrino species beyond the SM value.

comprised of axions.

The amount of dark matter generated in the early universe depends sensitively on the properties of the axino and the saxion in addition to the SUSY spectrum and the axion. For instance, thermally produced axinos can decay into LSPs after neutralino freeze-out thus augmenting the LSP abundance[126]. Saxions can be produced thermally or via coherent oscillations (important at large  $f_a$ ) and their decays can add to the LSP abundance, produce extra dark radiation in the form of axions or dilute all relics via entropy production from decays to SM particles[127]. The calculation of the mixed axion-WIMP abundance requires solution of eight coupled Boltzmann equations. Results from a mixed axion-higgsino dark matter calculation in natural SUSY are shown in Fig. 14[128]. At low  $f_a \sim 10^{10}$  GeV, then the thermal value of WIMP production is maintained since axinos decay before freeze-out. In this case the DM is axion-dominated[129]. For higher  $f_a$  values, then axinos and saxions decay after freeze-out thus augmenting the WIMP abundance. For very large  $f_a \gtrsim 10^{14}$  GeV, then WIMPs are overproduced and those cases would be excluded. Many of the high  $f_a$  models are also excluded via violations of BBN constraints and by overproduction of dark radiation- as parametrized by the effective number of extra neutrinos in the universe  $\Delta N_{eff}$ .

As far as dark matter detection goes, WIMP production in RNS was examined in Ref. [131]. There, it is emphasized that the relevant theory prediction for WIMP direct detection is the quantity  $\xi \sigma^{SI}(\tilde{Z}_1 p)$  where  $\xi = \Omega_{\tilde{Z}_1} h^2 / 0.12$  to reflect the possibility that the WIMP local abundance may be highly depleted, and perhaps axion-dominated. Nonetheless, WIMPs should be ultimately



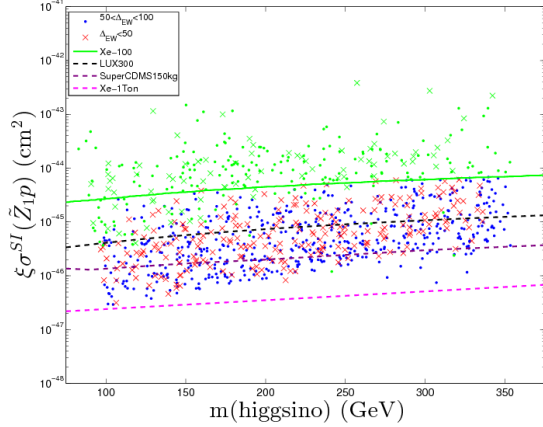


FIG. 15: Plot of rescaled higgsino-like WIMP spin-independent direct detection rate  $\xi\sigma^{SI}(\tilde{Z}_{1p})$  versus  $m(\text{higgsino})$  from a scan over NUHM2 parameter space with  $\Delta_{EW} < 50$  (red crosses) and  $\Delta_{EW} < 100$  (blue dots). Green points are excluded by current direct/indirect WIMP search experiments. We also show the current reach from Xe-100 experiment, and projected reaches of LUX, SuperCDMS 150 kg and Xe-1 ton. Plot from Ref. [131].

detected by ton-scale noble liquid detectors because naturalness insures that the WIMP-Higgs coupling— which is a product of higgsino and gaugino components— is never small (see Fig. 15). Prospects for indirect detection of higgsino-like WIMPs from WIMP-WIMP annihilations to gamma rays or anti-matter are less lucrative since then the expected detection rates must be scaled by  $\xi^2$ . Meanwhile, we would also expect ultimate detection of axions if natural SUSY prevails[132].

## CONCLUSIONS

The framework of supergravity gauge theories with SUSY breaking taking place in a hidden sector – as put forth by Arnowitt-Chamseddine-Nath and others[11] more than 30 years ago– provides a compelling and elegant picture for physics beyond the Standard Model. SUGRA gauge theories allow for a solution to the naturalness/hierarchy problem, allow for the inclusion of gravity into particle physics and provide a candidate for cold dark matter. They receive indirect support from the measured values of the gauge couplings, the top mass and the Higgs mass. In spite of these successes, they have come under rather severe criticism of late due to a (mis) perception of their increasing unnaturalness due to the rather high value of  $m_h$  and due to increasingly severe search limits from LHC. Opinions have been voiced that we are witnessing the downfall of one of the great paradigms of modern physics[133].

In this paper, we have refuted this point of view. We

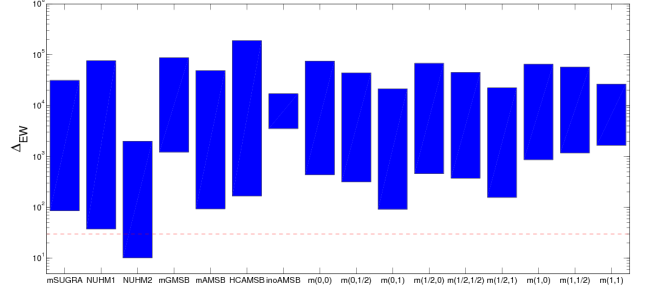


FIG. 16: Histogram of range of  $\Delta_{EW}$  values generated for each SUSY model considered in Ref. [50]: mSUGRA, NUHM1, NUHM2, mGMSB, mAMSB, HCAMSB, inoAMSB and various versions of mirage-mediation for different modular weight choices. We would consider  $\Delta_{EW} \lesssim 30$ — the lower the better— as acceptable values for EW fine-tuning. This region is located below the dashed red line.

noted that the oft-quoted, but seldom scrutinized, large-log measure of naturalness neglects dependent terms which allow for large cancellations in the contributions to the  $Z$  or Higgs mass. It is time for this measure to be set aside: sub-TeV top squarks are not required for SUSY naturalness.

The traditional BG measure of naturalness is almost always applied to the multi-parameter SUSY effective theories where independent soft terms are introduced to parametrize a vast array of hidden sector possibilities. If the soft terms of gravity-mediation are instead written as multiples of  $m_{3/2}$ , then their *dependence* is explicitly displayed and their contributions to  $m_Z$  or  $m_h$  can be properly combined. Thus, the BG measure is valid for SUGRA theories provided it is applied to Eq. 36. Once dependent terms are collected in their contributions to  $m_Z$  or  $m_h$ , both large-log and BG measures are seen to reduce to the electroweak measure  $\Delta_{EW}$ .

The naturalness criterion for low  $\Delta_{EW}$  is that the higgsino mass  $\mu$  and the weak scale soft term  $|m_{H_u}|$  are not too far from  $m_{Z,h}$ — in fact, the closer to  $m_Z$  the better. In addition, the top squarks can easily exist at the few TeV level so long as they are highly mixed by a large trilinear  $A_t$  term. This condition also lifts the Higgs mass to  $\sim 125$  GeV. These naturalness conditions are easily realized in the two-parameter non-universal Higgs model where  $m_{H_u}(GUT)$  is typically about 30% larger than  $m_0$ , the mass scale of the matter scalars. Then  $m_{H_u}^2$  is radiatively driven to negative values rather close to  $-m_Z^2$ . Such SUSY models contain radiatively-driven naturalness (RNS). Most other SUSY models which generate  $m_h \sim 125$  GeV are found to be *un-natural* (see Fig. 16[50]).

It is argued that naturalness should also be enforced in the QCD sector which leads to inclusion of the invisible axion. In the DFSZ SUSY axion model, the SUSY

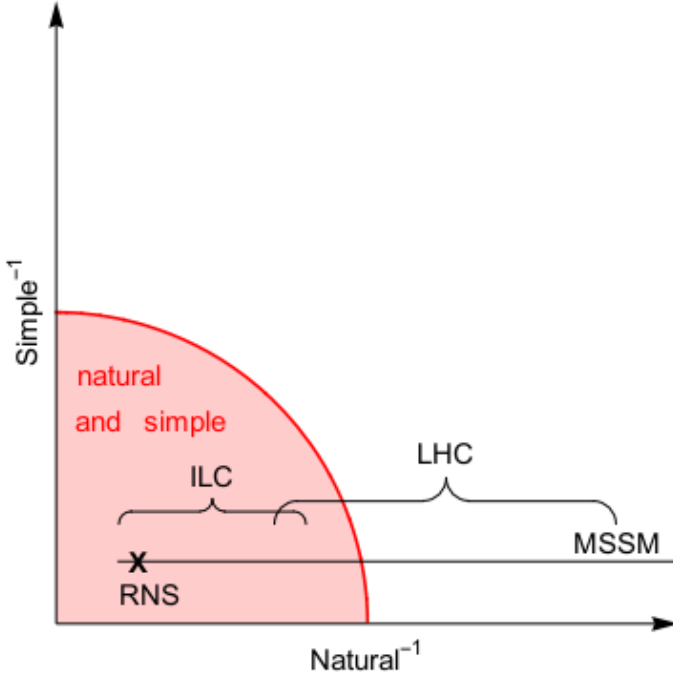


FIG. 17: Figurative plot of theory space in the  $1/\text{natural}$  vs.  $1/\text{simple}$  plane including the locus of the MSSM and the RNS SUSY models along with the approximate reach of LHC and ILC.

$\mu$  problem is elegantly solved. In fact, the  $\mu$  term can itself be generated such that  $\mu \ll m_{3/2}$  in a class of DFSZ SUSY axion models with radiatively broken PQ symmetry.

For RNS SUSY models, SUSY might be accessible to LHC searches but could also easily evade LHC searches with little cost to naturalness. The requisite light higgsino states, however, should be accessible to a linear  $e^+e^-$  collider operating with  $\sqrt{s} > 2m(\text{higgsino})$ . For RNS SUSY, we expect ultimate detection of both a higgsino-like WIMP and the axion. Discoveries such as these should vindicate the original vision put forth by Arnowitt-Chamseddine-Nath and others in their development of supergravity gauge theories.

Our ultimate plot is shown in Fig. 17 where we show a figurative plot of theory space in the  $1/\text{natural}$  vs.  $1/\text{simple}$  plane. The locus of the MSSM is along a line at high simplicity but extending from highly natural to highly un-natural. SUSY models with radiatively-driven naturalness (RNS) lie in the highly simple and highly natural regime. Future LHC searches will only be sensitive to a portion of the natural theory space. An ILC  $e^+e^-$  machine will be required to test for the presence of the required light higgsino states.

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